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Exam. Code : 103205
Subject Code : 8058

B.A./B.Sc. 5th Semester (Old Syllabus 2016)

MATHEMATICS

Paper—I (Vector Calculus & Solid Geometry)

Time Allowed—Three Hours] [Maximum Marks—50

Note :— Attempt any **FIVE** questions in all choosing at least **TWO** each from section.

SECTION—A

I. (a) Prove that a vector function $\vec{f}(t)$ has a constant magnitude if and only if :

$$\vec{f} \cdot \frac{\overrightarrow{df}}{dt} = \vec{0}.$$

(b) If $\vec{r}$ is a unit vector, then prove that :

$$\left| \vec{r} \times \frac{\overrightarrow{dr}}{dt} \right| = \left| \frac{\overrightarrow{dr}}{dt} \right|. \qquad 5,5$$

II. (a) Prove that grad V is a vector normal to the surface $V(x, y, z) = c$, where c is a constant.

(b) Prove that :

$$\text{div}\left(\frac{f(r)}{r}\vec{r}\right) = \frac{1}{r^2}\frac{d}{dr}[r^2 f(r)]. \qquad 5,5$$

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III. (a) Discuss physical interpretation of curl of a vector point function.

(b) For any vector function $\vec{v}$, prove that :

$$\text{grad div } \vec{v} = \text{curl curl } \vec{v} + \nabla^2 \vec{v}. \qquad 5,5$$

IV. (a) If $\vec{F} = 3xy\hat{i} - y^2\hat{j}$, evaluate $\int_C \vec{F}.d\vec{r}$, where C is the curve in the x – y plane, $y = 2x^2$ from (0, 0) to (1, 2).

(b) State and prove Gauss's Divergence Theorem. 2,8

V. (a) Verify Stoke's theorem for $\vec{F} = (y - \sin x)\hat{i} + \cos x\,\hat{j}$ over the triangle with vertices (0, 0), $\left(\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, 1\right)$.

(b) Prove that :

$$\oint \vec{r}.d\vec{r} = 0. \qquad 8,2$$

SECTION—B

VI. (a) Trace the locus of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{2z}{c}$, where a, b, c are positive.

(b) Obtain the equation of the surface of revolution obtained by rotating the curve $y^2 = 4ax$, $z = 0$ about the x-axis. 7,3

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VII. (a) Reduce $x^2 - 4xy + 4y^2 - 32x + 4y + 16 = 0$ to the standard form and identify it.

(b) Find the condition that the plane $lx + my + nz = p$ may touch the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. 5,5

VIII. (a) Prove that the sum of the squares of the reciprocals of any three mutually perpendicular diameters of an ellipsoid is constant.

(b) Find the equation of the tangent plane at the point (x', y', z') of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. Show that the length p of the perpendicular from the origin on the tangent plane at the point (x', y', z') is given by :

$$\frac{1}{p^2}=\frac{x'^2}{a^4}+\frac{y'^2}{b^4}+\frac{z'^2}{c^4}.$$ 4,6

IX. (a) Prove that there are six points on an ellipsoid the normals at which pass through a given point (α, β, γ).

(b) Find the locus of points from which three mutually perpendicular tangents can be drawn to the paraboloid $ax^2 + by^2 = 4z$. 5,5

X. (a) Determine the centre of the conicoid $F(x, y, z) = 0$.

(b) Prove that the surface whose equation is :

$$x^2 + 6y^2 - z^2 - yz + 5xy + 2x + 5y = 0$$

represents hyperbolic cylinder. 2,8

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